BAULKHAM HILLS HIGH SCHOOL

**2008**

**TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION**

# Mathematics

**General Instructions**

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question
- Marks may be deducted for careless or badly arranged work

**Total marks – 120**

- Attempt Questions 1 – 10
- All questions are of equal value
- Start a separate piece of paper for each question.
- Put your student number and the question number at the top of each sheet.

**Total marks – 120**
**Attempt Questions 1 – 10**
**All questions are of equal value**

Answer each question on a SEPARATE piece of paper clearly marked Question 1, Question 2, etc. Each piece of paper must show your student number.

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**Question 1** ***(12 marks)*** Use a *separate* piece of paper ***Marks***

a) Evaluate $\sqrt{e^3+3}$ correct to three decimal places. *2*

b) Factorise $x^3+64$ *2*

c) Differentiate $x^2+\ln x$ *2*

d) When GST of 10% is added on, the selling price of a laptop becomes \$1375. What was the price of the laptop before the GST was added? *2*

e) Find the values of *x* for which $|5-2x|\le 3$ *2*

f) Given that the equation $kx^2+12x+k=0$, where *k* is a positive constant, has equal roots, find the value of *k*. *2*

**Question 2** ***(12 marks)*** Use a *separate* piece of paper

a) Differentiate with respect to *x*:

(i) $xe^{2x}$ *2*

(ii) $\dfrac{x}{\cos x}$ *2*

b) (i) $\int \dfrac{x^2}{x^3+2}\,dx$ *2*

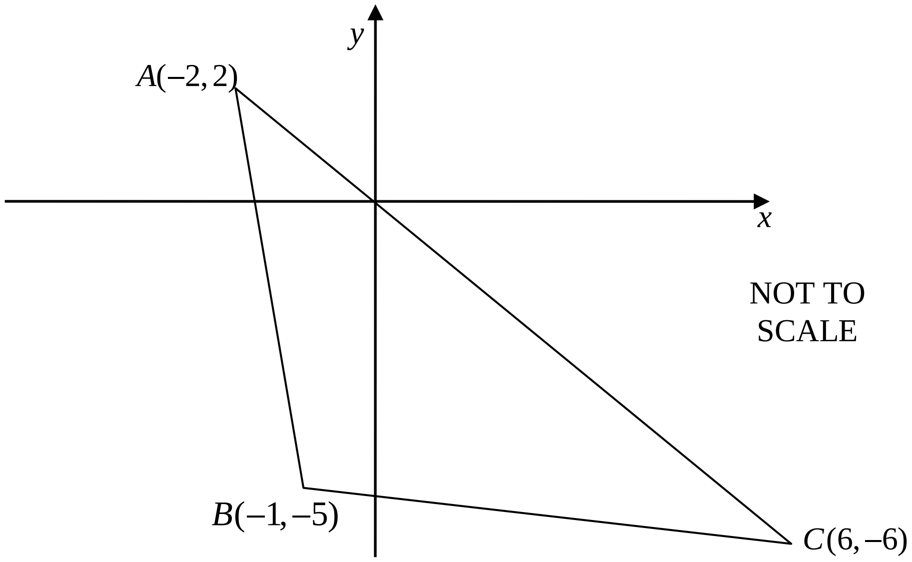
(ii) $\int_0^{\frac{\pi}{4}} \left(\sin 2x+\sec^2 x\right)dx$ *3*

c) Find the equation of the normal to the curve $y=2\sin x+1$ at the point $(\pi,1)$. *3*

**Question 3** ***(12 marks)*** Use a *separate* piece of paper **_Marks_**

a) Solve $\sqrt{3}\tan x + 1 = 0$ for $0 \le x \le 2\pi$. *2*

b)




In the diagram *A*, *B* and *C* are the points $(-2, 2)$, $(-1, -5)$ and $(6, -6)$ respectively. Copy or trace this diagram onto your answer sheet.

(i) Show that the midpoint *P* of *AC* has coordinates $(2, -2)$. *1*

(ii) Find the gradient of *BP*. *1*

(iii) Show that $BP \perp AC$. *2*

(iv) Find the coordinates of *D* if *P* is the midpoint of the interval *BD*. *2*

(v) What kind of quadrilateral is *ABCD*? *1*

(vi) Show that the diagonal *AC* has length $8\sqrt{2}$ units. *1*

(vii) Find the area of the quadrilateral *ABCD*. *2*

**Question 4** ***(12 marks)*** Use a *separate* piece of paper

a) Solve $\log_3(2x+3) = 4$. *2*

b) Pocholo repays a loan over a period of *n* months. He repays \$149 in the first month, \$147 in the second month, \$145 in the third month, and so on.

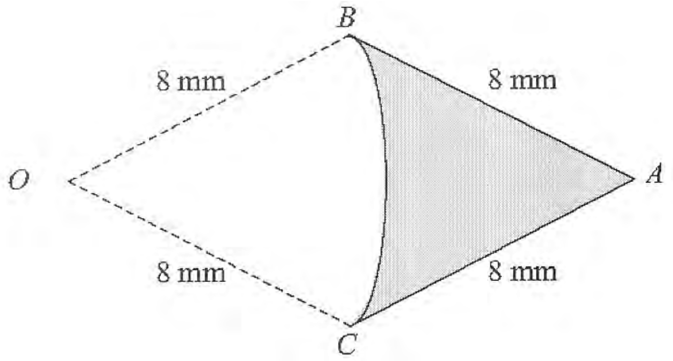
(i) How much does Pocholo repay in the $21^{st}$ month? *2*

(ii) How much in total has Pocholo repaid in the first 21 months? *1*

(iii) Pocholo repays a total of \$5000. How many repayments does he make? *2*

**Question 4** ***(continued)*** ***Marks***

c)




In the diagram, the shaded area *ABC* represents a badge where *AB* = *AC* = 8 mm.
The curve *BC* is an arc of a circle, centre *O* and radius 8 mm.

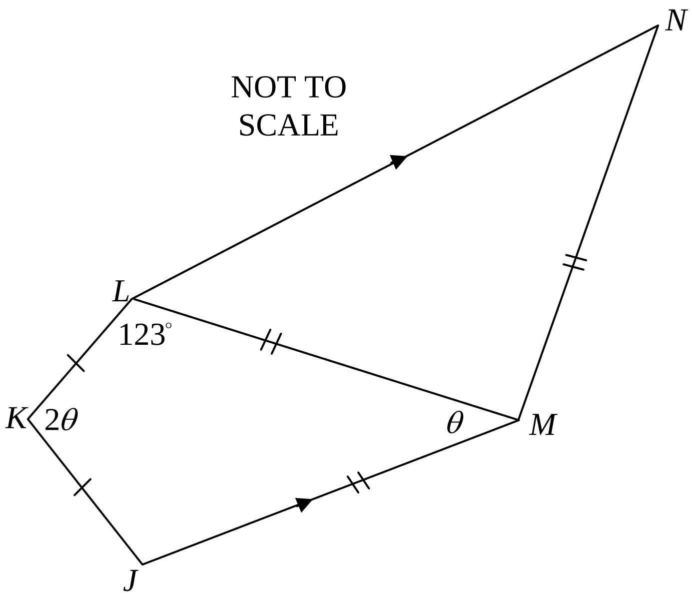
$\angle BAC = \angle BOC = \dfrac{2\pi}{5}$ radians.

(i) Using the Cosine Rule, find the distance of the interval *BC*, correct to one decimal place. *2*

(ii) Find the area of the badge, correct to one decimal place. *3*

**Question 5** ***(12 marks)*** Use a *separate* piece of paper

a)




In the diagram *JKLM* is a quadrilateral and *LMN* is a triangle.
$JM \parallel LN$, $JK = KL$, $JM = ML = MN$, $\angle KLM = 123^\circ$, $\angle JKL = 2\theta$ and $\angle JML = \theta$.
Copy or trace this diagram onto your answer sheet.

(i) Prove $\Delta KLM \equiv \Delta KJM$. *2*

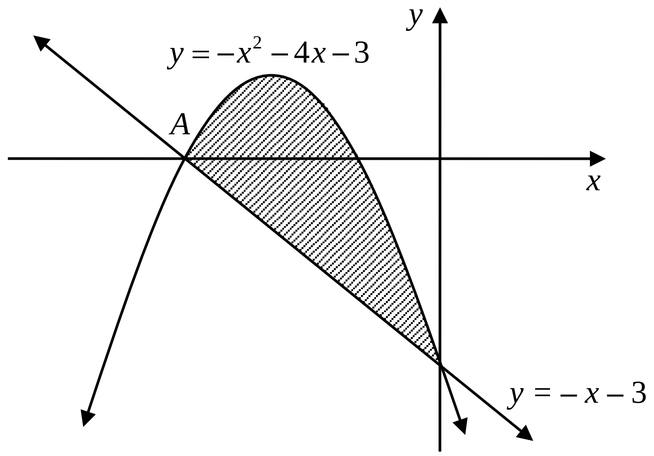
(ii) Explain why $\angle KLM = \angle KJM$. *1*

(iii) Show that $\angle JML = 38^\circ$, giving reasons. *1*

(iv) Find the size of $\angle LNM$ , giving reasons *2*

**<u>Question 5</u>** ***(continued)*** ***Marks***

b)




The diagram shows the sketch of the parabola $y=-x^2-4x-3$ and a line $y=-x-3$

(i) Find the *x* coordinate of *A*. *1*

(ii) Find the area of the shaded region bounded by the line and the parabola. *2*

c) Samantha has three similar keys in her pocket. To open her front door she tried the keys at random. She stopped trying when she opened the door and she did not use the same key twice.
Find the probability that;

(i) the door opened when she tried the first key. *1*

(ii) she tried all three keys before the door opened. *2*

**<u>Question 6</u>** ***(12 marks)*** Use a *separate* piece of paper

a) Consider the function $f(x)=9x(x-2)^2$.

(i) Find the coordinates of the stationary points of the curve $y=f(x)$ and determine their nature. *3*

(ii) Find the coordinates of the point of inflection. *1*

(iii) Sketch the curve $y=f(x)$ showing where it meets the axes. *2*

(iv) Find the values of *x* for which the curve $y=f(x)$ is concave up. *1*

**Question 6** ***(continued)*** ***Marks***

b) Wheat is poured from a silo into a truck at a rate of $\frac{dM}{dt}$ kilograms per second, where $\frac{dM}{dt} = 81t - t^3$ and $t$ is the time in seconds after the wheat begins to flow.

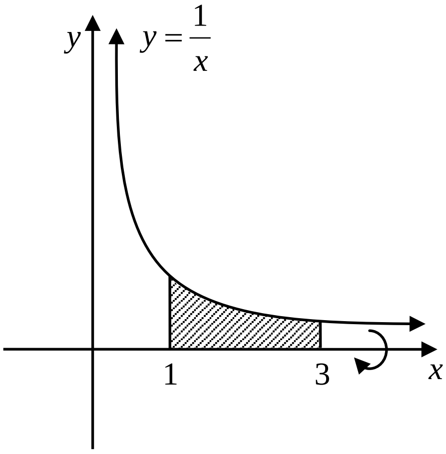
(i) Find an expression for the mass $M$ kg of wheat in the truck after $t$ seconds, If initially there was 1 tonne (1000 kg) of wheat in the truck. *2*

(ii) Calculate the total weight of wheat in the truck after 6 seconds. *1*

(iii) What is the largest value of $t$ for which the expression for $\frac{dM}{dt}$ is physically possible? *2*

**Question 7** ***(12 marks)*** Use a *separate* piece of paper

a) *3*




In the diagram, the shaded region is bounded by the curve $y = \frac{1}{x}$, the lines $x = 1$ and $x = 3$, and the $x$-axis. The shaded region is rotated about the $x$-axis. Calculate the exact volume of the solid of revolution formed.

b) A particle is moving on the $x$-axis. Its velocity, $v$ metres per second, at time $t$ seconds is given by $v = 3 - 6\cos t$.

(i) What is the maximum velocity of the particle? *2*

(ii) When does the particle first come to rest? *2*

(iii) Sketch the graph of $v$ as a function of $t$ for $0 \le t \le 2\pi$ *2*

(iv) Calculate the total distance travelled by the particle between $t = 0$ and $t = \pi$. *3*

**<u>Question 8</u>** ***(12 marks)*** Use a *separate* piece of paper **_Marks_**

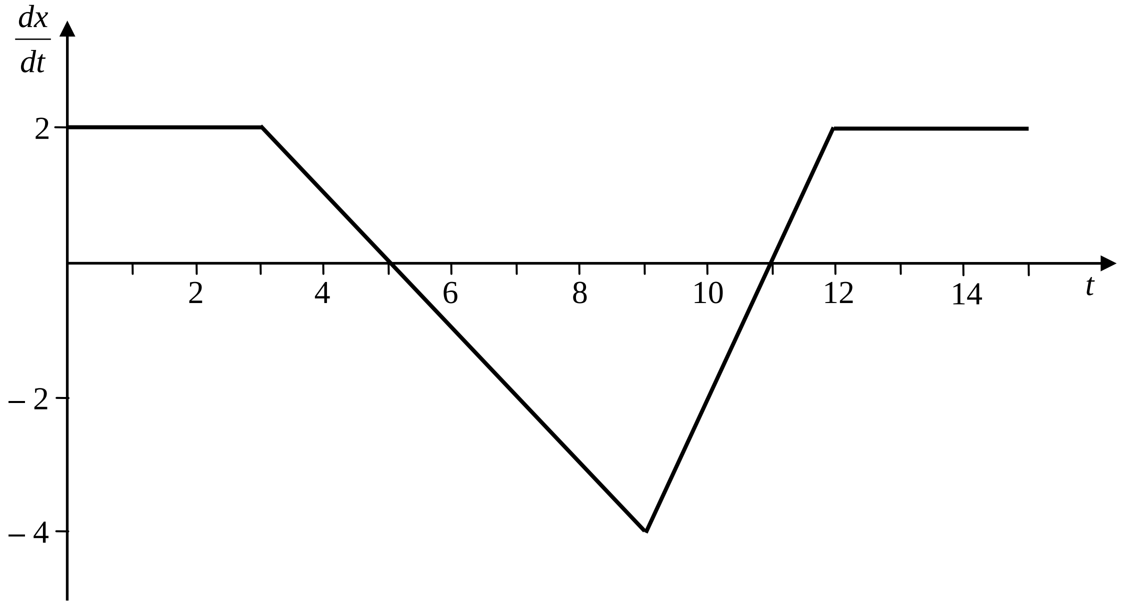
a) The amount $N$ grams of a given carbon isotope in a dead tree trunk is given by $N = Ae^{-kt}$ where $A$ and $k$ are positive constants.

(i) Show that $N$ satisfies the equation $\dfrac{dN}{dt} = -kN$. *1*

(ii) It is estimated that the tree trunk originally contained 500 grams of the carbon isotope and 300 grams remain after 500 years. Find $A$ and $k$. *3*

(iii) The tree trunk now contains 100 grams of the carbon isotope. How long ago did the tree die? Give your answer to the nearest 100 years. *2*

b)




The graph shows the velocity, $\dfrac{dx}{dt}$, of a particle as a function of time, in its first 15 seconds of motion. Initially the particle is at the origin.

(i) At what time does the particle first return to the origin? Justify your answer. *2*

(ii) What is the maximum displacement of the particle, and when does it occur? *2*

(iii) Draw a sketch of the particle's displacement, $x$, as a function of time. *2*

**<u>Question 9</u>** ***(12 marks)*** Use a *separate* piece of paper **Marks**

a) At the beginning of the year 2008 a company bought a new machine for $15000. Each subsequent year the value of the machine decreases by 20% of its value at the start of the year. When the value of the machine falls below $500, the company will replace it.

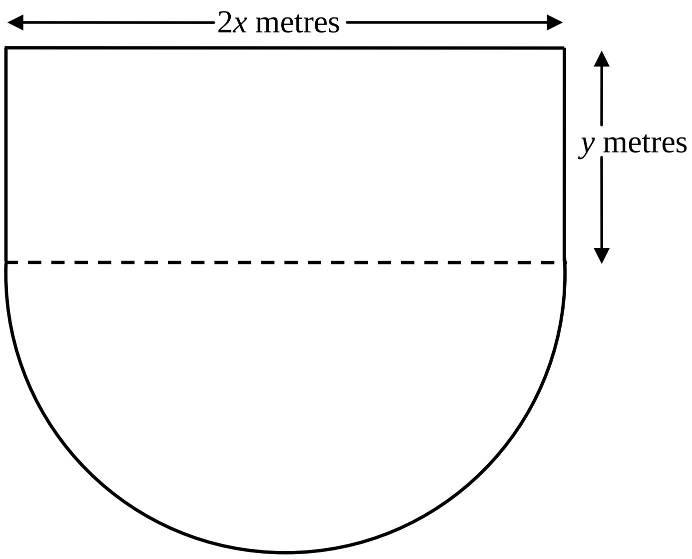
To plan for a replacement machine, the company pays $1000 at the start of each year into a savings account. The account pays a fixed rate of 5% compound interest p.a. The first payment was made when the machine was bought and the last payment will be made at the start of the year in which the machine is replaced.

(i) What will be the value of the machine at the start of 2010? *1*

(ii) In which year will the company replace the machine? *3*

(iii) How much will the savings account be worth at the end of the year in which the machine will be replaced? *3*

b)




The diagram shows the plan of a stage in the shape of a rectangle joined to a semicircle. The diameter of the semicircle is $2x$ metres and the width of the rectangle is $y$ metres. The perimeter of the stage is 80 metres.

(i) Show that the area, $A$ m$^2$, of the stage is given by $A = 80x - \left(2 + \frac{\pi}{2}\right)x^2$ *2*

(ii) Hence, or otherwise, show that the stage has a maximum area when *3*

$$x = \frac{80}{\pi + 4}$$

**<u>Question 10</u>** ***(12 marks)*** Use a *separate* piece of paper **_Marks_**

a) (i) Show that $\int_0^2 \frac{dx}{1+x} = \ln 3$ *1*

(ii) Use Simpson's Rule with five function values to find an approximation to $\ln 3$. *2*

b) $\alpha$ and $\beta$ are the roots of the equation $x^2 + 4x + 2 = 0$.

(i) Find the value of $\alpha^2 + \beta^2$. *2*

(ii) Hence, or otherwise, write down the quadratic equation whose roots are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$. *3*

c) It is known that the geometric series $1 + x + x^2 + x^3 + \ldots$ has a limiting sum.

(i) What are the possible values of *x?* *1*

(ii) Observe that; *3*

$$1 = 1$$
$$2x = x + x$$
$$3x^2 = x^2 + x^2 + x^2$$
$$4x^3 = x^3 + x^3 + x^3 + x^3$$

By studying the above arrangement, or otherwise, find in simplest algebraic form, an expression for the limiting sum of the series;

$$1 + 2x + 3x^2 + 4x^3 + \ldots + nx^{n-1} + \ldots$$

**END OF PAPER**

# STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \; n \neq -1; x \neq 0, \text{ if } \; n < 0$$

$$\int \frac{1}{x} dx = \ln x, \; x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \; a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \; a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \; a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \; a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \; a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \; a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \; a > 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \; x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

**NOTE:** $\ln x = \log x, \quad x > 0$

# Baulkham Hills Mathematics Trial 2008 Solutions

## Question 1 (12)

a) $\sqrt{e^3+3} = 4.804741088\ldots$
$= 4.805$ (to 3dp) (2)

b) $x^3+64$
$= (x+4)(x^2-4x+16)$ (2)

c) $f(x) = x^2 + \log x$
$f'(x) = 2x + \frac{1}{x}$ (2)

d) $110\% = \$1375$
$100\% = \frac{1375}{110} \times 100$
$= \$1250$ (2)

e) $|5-2x| \le 3$
$5-2x \le 3$ or $-(5-2x) \le 3$
$-2x \le -2$ $\quad -5+2x \le 3$
$x \ge 1$ — 1 — $2x \le 8$
$x \le 4$
$\therefore 1 \le x \le 4$ 1 (2)

f) equal roots occur when $\Delta = 0$
ie $12^2 - 4(k)(k) = 0$
$144 - 4k^2 = 0$
$4k^2 = 144$
$k^2 = 36$ 1
$k = 6$ 1 $(\because k > 0)$ (2)

## Question 2 (12)

a) $f(x) = xe^{2x}$
$f'(x) = (x)(2e^{2x}) + (e^{2x})(1)$ 1
$= 2xe^{2x} + e^{2x}$ 1 (2)
$= (2x+1)e^{2x}$

(ii) $f(x) = \frac{x}{\cos x}$
$f'(x) = \frac{(\cos x)(1) - (x)(-\sin x)}{\cos^2 x}$ 1
$= \frac{\cos x + x\sin x}{\cos^2 x}$ 1 (2)

b) $\int \frac{x^2}{x^3+2}\,dx = \frac{1}{3}\int \frac{3x^2}{x^3+2}\,dx$ (2)
$= \frac{1}{3}\log(x^3+2) + c$ 1

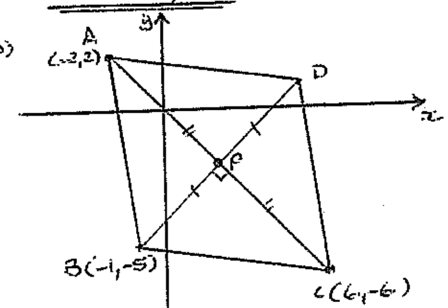
(ii) $\int_0^{\frac{\pi}{4}} (\sin 2x + \sec^2 x)\,dx$
$= \left[-\frac{1}{2}\cos 2x + \tan x\right]_0^{\frac{\pi}{4}}$ 1
$= -\frac{1}{2}\cos\frac{\pi}{2} + \tan\frac{\pi}{4} + \frac{1}{2}\cos 0 - \tan 0$ 1
$= 0 + 1 + \frac{1}{2} - 0$
$= \frac{3}{2}$ 1 (3)

c) $y = 2\sin x + 1$
$\frac{dy}{dx} = 2\cos x$ 1
when $x = \pi$, $\frac{dy}{dx} = 2\cos\pi$
$= -2$
$\therefore$ required slope $= \frac{1}{2}$ 1
$y - 1 = \frac{1}{2}(x - \pi)$
$2y - 2 = x - \pi$
$x - 2y + 2 - \pi = 0$ 1 (3) — as $y = mx+b$ or general form

## Question 3 (12)

a) $\sqrt{3}\tan x + 1 = 0 \quad 0 \le x \le 2\pi$
$\tan x = -\frac{1}{\sqrt{3}}$
Q2, 4
$\tan\alpha = \frac{1}{\sqrt{3}}$
$\alpha = \frac{\pi}{6}$
$x = \pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$
$x = \frac{5\pi}{6}$ 1, $\frac{11\pi}{6}$ 1 (2)

b)




(i) $P = \left(\frac{-2+6}{2}, \frac{2-6}{2}\right)$
$= \left(\frac{4}{2}, \frac{-4}{2}\right)$
$= (2, -2)$ (1)

(ii) $m_{BP} = \frac{-5+2}{-1-2}$
$= \frac{-3}{-3}$
$= 1$ (1)

(iii) $m_{AC} = \frac{2+6}{-2-6}$
$= \frac{8}{-8}$ 1
$= -1$
$m_{BP} \times m_{AC} = 1 \times -1$
$= -1$ 1
$\therefore BP \perp AC$ (2)

(iv) $\frac{-1+x}{2} = 2 \qquad \frac{-5+y}{2} = -2$
$-1+x = 4 \qquad -5+y = -4$
$x = 5$ 1 $\qquad y = 1$ 1 (2)
$\therefore D$ is $(5,1)$

(v) diagonals bisect at right angles
$\therefore ABCD$ is a rhombus (1)

(vi) $d_{AC} = \sqrt{(6+2)^2 + (-6-2)^2}$
$= \sqrt{64+64}$
$= \sqrt{128}$
$= 8\sqrt{2}$ units (1)

(vii) $d_{BD} = \sqrt{(-1-5)^2 + (-5-1)^2}$
$= \sqrt{36+36}$
$= 6\sqrt{2}$ units 1
$A = \frac{1}{2} \times 8\sqrt{2} \times 6\sqrt{2}$
$= 48$ units$^2$ 1 (2)

## Question 4 (12)

a) $\log_3(2x+3) = 4$
$2x + 3 = 3^4$ 1
$= 81$
$2x = 78$
$x = 39$ 1 (2)

b) $a = 149 \quad d = -2$
$T_{21} = a + 20d$ 1
$= 149 + 20(-2)$
$= \$109$ 1 (2)

(ii) $S_{21} = \frac{21}{2}\{2a + 20d\}$
$= 21(a + 10d)$
$= 21[149 + 10(-2)]$
$= 21 \times 129$
$= \$2709$ (1)

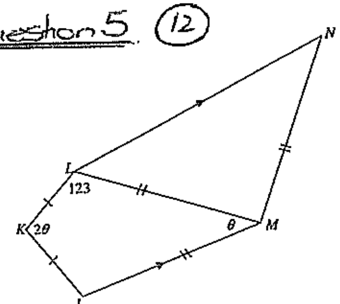
(iii) $S_n = 5000$
$\frac{n}{2}\{2a + (n-1)d\} = 5000$
$\frac{n}{2}\{2(149) + (n-1)(-2)\} = 5000$
$n(149 - n + 1) = 5000$
$150n - n^2 = 5000$ 1
$n^2 - 150n + 5000 = 0$
$(n-50)(n-100) = 0$
$n = 50$ or $n = 100$ (2)
$\therefore$ He makes 50 repayments. 1

c) $BC^2 = 8^2 + 8^2 - 2 \times 8 \times 8 \times \cos\frac{2\pi}{5}$ 1
$BC = 9.404564037\ldots$
$BC = 9.4$ mm (to 1dp) 1 (2)

(ii) Area $= 2 \times \frac{1}{2} \times 8 \times 8 \times \sin\frac{2\pi}{5}$ 1
$- \frac{1}{2} \times 8^2 \times \frac{2\pi}{5}$ 1
$= 20.65523108$
$= 20.7$ mm$^2$ (to 1dp) 1 (3)

## Question 5 (12)

a)




(i) $KL = KJ$ (given)
$ML = MJ$ (given) 1
$MK$ is common
$\therefore \triangle KLM \equiv \triangle KJM$ 1 (SSS) (2)

(ii) $\angle KLM = \angle KJM$
(matching $\angle$'s in $\equiv \triangle$'s) (1)

(iii) $\angle JKL + \angle KLM + \angle LMJ + \angle MJK = 360°$ ($\angle$ sum KJML)

$2\theta + 123 + \theta + 123 = 360$

$3\theta = 114$

$\theta = 38$

$\therefore \angle JML = 38°$ (1)

(iv) $\Delta LMN$ is isosceles (LM = MN, given)

$\therefore \angle LNM = \angle MLN$ (base $\angle$'s, isosceles $\Delta$)

$\angle NLM = \angle LMJ$ (alternate $\angle$'s, $LN \parallel JM$)

$\therefore \angle LNM = \angle LMJ$

$\angle LNM = 38°$ (2)

b) $-x^2 - 4x - 3 = -x - 3$

$x^2 + 3x = 0$

$x(x+3) = 0$

$x = 0$ or $x = -3$

$\therefore$ x coordinate of A is $-3$ (1)

(ii) $A = \int_{-3}^{0} [(-x^2-4x-3)-(-x-3)]\,dx$

$= \int_{-3}^{0} (-x^2 - 3x)\,dx$

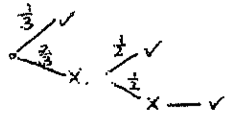
$= \left[-\frac{1}{3}x^3 - \frac{3}{2}x^2\right]_{-3}^{0}$

$= -\frac{1}{3}(0)^3 - \frac{3}{2}(0)^2 - \frac{1}{3}(3)^3 + \frac{3}{2}(3)^2$

$= 4.5$ units² (2)

c) P(1st key open) $= \frac{1}{3}$ (1)

(ii)



P(all 3 keys) $= \frac{2}{3} \times \frac{1}{2} \times 1$

$= \frac{1}{3}$ (2)

## Question 6 (12)

a) $f(x) = 9x(x-2)^2$

$= 9x(x^2 - 4x + 4)$

$= 9x^3 - 36x^2 + 36x$

$f'(x) = 27x^2 - 72x + 36$

$f''(x) = 54x - 72$

$f'''(x) = 54$

(i) stationary pts occur when $f'(x) = 0$

ie $27x^2 - 72x + 36 = 0$

$3x^2 - 8x + 4 = 0$

$3x^2 - 6x - 2x + 4 = 0$

$3x(x-2) - 2(x-2) = 0$

$(x-2)(3x-2) = 0$

$x = 2$ or $x = \frac{2}{3}$

$\therefore$ stationary pts are $\left(\frac{2}{3}, \frac{32}{3}\right)$ and $(2, 0)$

when $x = \frac{2}{3}$, $f''\left(\frac{2}{3}\right) = 54\left(\frac{2}{3}\right) - 72 = -36 < 0$

$\therefore \left(\frac{2}{3}, \frac{32}{3}\right)$ is a maximum tp

when $x = 2$, $f''(2) = 54(2) - 72 = 36 > 0$

$\therefore (2, 0)$ is a minimum tp (3)

(ii) possible points of inflection occur when $f''(x) = 0$

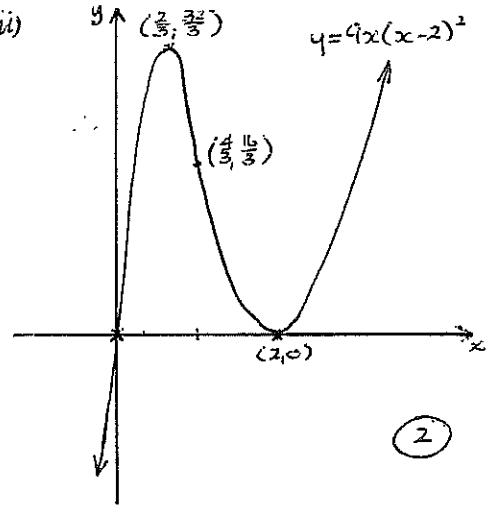
ie $54x - 72 = 0$

$x = \frac{4}{3}$

when $x = \frac{4}{3}$, $f'''\left(\frac{4}{3}\right) = 54 \neq 0$

$\therefore \left(\frac{4}{3}, \frac{16}{3}\right)$ is an inflection pt (1)

(iii)




(2)

(iv) curve is concave up

$x > \frac{4}{3}$ (1)

b) $\frac{dM}{dt} = 81t - t^3$

$M = \frac{81}{2}t^2 - \frac{1}{4}t^4 + c$

when $t = 0$, $M = 1000$

ie $1000 = 0 - 0 + c$

$c = 1000$

$\therefore M = \frac{81}{2}t^2 - \frac{1}{4}t^4 + 1000$ (2)

(ii) when $t = 6$,

$M = \frac{81}{2}(6)^2 - \frac{1}{4}(6)^4 + 1000$

$= 2134$

$\therefore$ after 6 seconds there are 2134 kg of wheat in the truck (1)

(iii) $\frac{dM}{dt} = 0$

$81t - t^3 = 0$

$t(81 - t^2) = 0$

$t = 0$ or $t = \pm 9$

$\therefore$ wheat stops flowing after 9 seconds

| $t$ | $9^-$ (6) | 9 | $9^+$ (12) |
|---|---|---|---|
| $\frac{dM}{dt}$ | 270 | 0 | −756 |
| | / | — | \ |

after 9 seconds the mass is decreasing, which is not possible

$\therefore$ largest value of $t$ for which $\frac{dM}{dt}$ is physically possible is 9 seconds. (2)

## Question 7 (12)

a) $V = \pi\int_1^3 y^2\,dx$

$= \pi\int_1^3 \frac{1}{x^2}\,dx$

$= \pi\int_1^3 x^{-2}\,dx$

$= -\pi\left[x^{-1}\right]_1^3$

$= -\pi\left(\frac{1}{3} - 1\right)$

$= \frac{2\pi}{3}$ units³ (3)

b) $v = 3 - 6\cos t$ (1 for working)

amplitude $= 6$ (2)

$\therefore$ maximum velocity is 9 m/s

(ii) particle comes to rest when $v = 0$

ie $3 - 6\cos t = 0$

$6\cos t = 3$

$\cos t = \frac{1}{2}$

Q1, 4

$\cos\alpha = \frac{1}{2}$

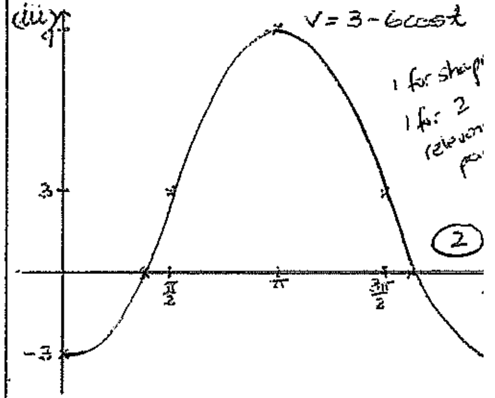
$\alpha = \frac{\pi}{3}$

$t = \alpha, 2\pi - \alpha$

$t = \frac{\pi}{3}, \frac{5\pi}{3}$ (2)

$\therefore$ particle first comes to rest after $\frac{\pi}{3}$ seconds

(iii)




1 for shape, 1 for 2 relevant points (2)

(iv) distance $= -\int_0^{\frac{\pi}{3}}(3 - 6\cos t)\,dt + \int_{\frac{\pi}{3}}^{\pi}(3 - 6\cos t)\,dt$

$= -\left[3t - 6\sin t\right]_0^{\frac{\pi}{3}} + \left[3t - 6\sin t\right]_{\frac{\pi}{3}}^{\pi}$

$= -3\left(\frac{\pi}{3}\right) + 6\sin\frac{\pi}{3} + 0 + 3(\pi) - 6\sin\pi - 3\left(\frac{\pi}{3}\right) + 6\sin\frac{\pi}{3}$

$= -6\left(\frac{\pi}{3}\right) + 12\sin\frac{\pi}{3} + 3\pi$

$= -2\pi + 6\sqrt{3} + 3\pi$

$= \pi + 6\sqrt{3}$ metres (3)

## Question 8 (12)

a) (i) $N = Ae^{-kt}$

$\frac{dN}{dt} = -Ake^{-kt}$

$= -kN$ (1)

(ii) when $t = 0$, $N = 500$

ie $500 = Ae^{0}$

$A = 500$ (1)

$\therefore N = 500e^{-kt}$

when $t = 500$, $N = 300$

ie $300 = 500e^{-500k}$

$e^{-500k} = \frac{3}{5}$ 1

$-500k = \log\frac{3}{5}$

$k = -\frac{1}{500}\log\frac{3}{5}$ 1 (2)

or

$= \frac{1}{500}\log\frac{5}{3}$

(iii) $N = 100$

$\therefore 100 = 500e^{-kt}$

$e^{-kt} = \frac{1}{5}$

$-kt = \log\frac{1}{5}$ 1

$t = -\frac{1}{k}\log\frac{1}{5}$

$= \frac{500\log\frac{1}{5}}{\log\frac{3}{5}}$

$= 1575.330052\ldots$ 1 (2)

$\therefore$ the tree died approximately 1600 years ago

b) Displacement is area under the curve.

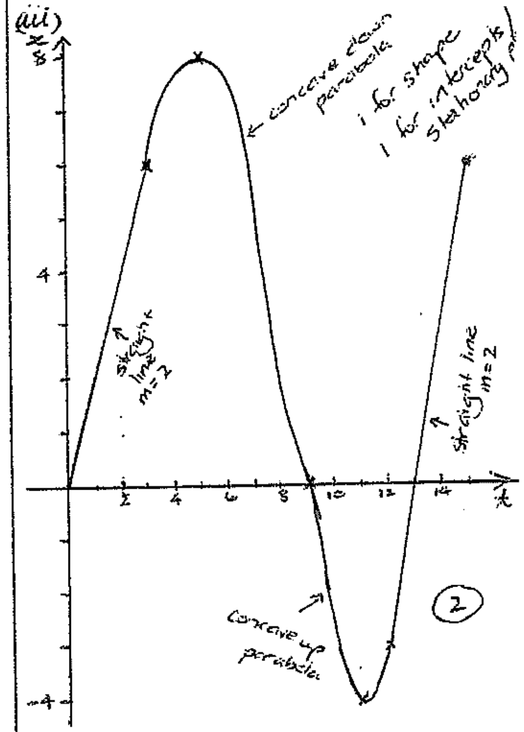
$t = 0$ to $3$, Area $= 6$ $\therefore x = 6$
$t = 3$ to $5$, Area $= 2$ $\therefore x = 8$
$t = 5$ to $9$, Area $= -8$ $\therefore x = 0$
$t = 9$ to $11$, Area $= -4$ $\therefore x = -4$
$t = 11$ to $12$, Area $= 1$ $\therefore x = -3$
$t = 12$ to $15$, Area $= 6$ $\therefore x = 3$

(i) Returns to origin when area above = area below 1

$\therefore$ returns to origin after 9 seconds 1 (2)

(ii) Maximum displacement is $x = 8$ which occurs after 5 seconds. 1 1 (2)

(iii)




## Question 9 (12)

a) $A_n = 15000(0.8)^n$

$A_2 = 15000(0.8)^2$

$= 9600$ 1

$\therefore$ machine will be worth \$9600 at start of 2010 (1)

(ii) $A_n < 500$

$15000(0.8)^n < 500$

$(0.8)^n < \frac{1}{30}$

$\log(0.8)^n < \log\frac{1}{30}$

$n\log 0.8 < \log\frac{1}{30}$

$n > \frac{\log\frac{1}{30}}{\log 0.8}$ 2

$n > 15.24219437$

$\therefore$ company will replace the machine during 2023 1 (3)

(iii) 1st investment $= 1000(1.05)^{16}$

2nd investment $= 1000(1.05)^{15}$

$\vdots$

last investment $= 1000(1.05)$ 1

Total $= 1000\{1.05 + 1.05^2 + \ldots + 1.05^{16}\}$

$a = 1.05$, $r = 1.05$, $n = 16$

$= 1000\left[\frac{1.05(1.05^{16} - 1)}{0.05}\right]$ 1

$= 24840.36636$ 1 (3)

$\therefore$ Account will be worth \$24840.37

b) $P = 80$

$2x + 2y + \pi x = 80$

$2y = 80 - 2x - \pi x$ 1

$A = 2xy + \frac{1}{2}\pi x^2$

$= x(80 - 2x - \pi x) + \frac{1}{2}\pi x^2$

$= 80x - 2x^2 - \pi x^2 + \frac{1}{2}\pi x^2$

$= 80x - 2x^2 - \frac{1}{2}\pi x^2$

$A = 80x - \left(2 + \frac{\pi}{2}\right)x^2$ 1 (2)

(ii) $\frac{dA}{dx} = 80 - 2\left(2 + \frac{\pi}{2}\right)x$

$= 80 - (4 + \pi)x$ 1

$\frac{d^2A}{dx^2} = -(4 + \pi)$

stationary pts occur when $\frac{dA}{dx} = 0$

ie $80 - (4 + \pi)x = 0$ 1

$x = \frac{80}{4 + \pi}$

when $x = \frac{80}{4+\pi}$, $\frac{d^2A}{dx^2} = -(4 + \pi) < 0$ 1 (3)

$\therefore$ when $x = \frac{80}{4+\pi}$ the area is a maximum.

## Question 10 (12)

a) (i) $\int_0^2 \frac{dx}{1+x} = \left[\log(1+x)\right]_0^2$ 1

$= \log 3 - \log 1$

$= \log 3$ (1)

(ii)

| | 1 | 4 | 2 | 4 | 1 |
|---|---|---|---|---|---|
| $x$ | 0 | $\frac{1}{2}$ | 1 | $\frac{3}{2}$ | 2 |
| $\frac{1}{1+x}$ | 1 | $\frac{2}{3}$ | $\frac{1}{2}$ | $\frac{2}{5}$ | $\frac{1}{3}$ |

$\int_0^2 \frac{dx}{1+x} \doteqdot \frac{h}{3}\{y_0 + 4y_{odd} + 2y_{even} + y_n\}$

$= \frac{\frac{1}{2}}{3}\left\{1 + 4\left(\frac{2}{3} + \frac{2}{5}\right) + 2\left(\frac{1}{2}\right) + \frac{1}{3}\right\}$ 1

$= \frac{11}{10}$

$= 1.1$ 1 (2)

$\therefore \log 3 \doteqdot 1.1$

b) (i) $x^2 + 4x + 2 = 0$

$\alpha + \beta = -4$ $\quad \alpha\beta = 2$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (-4)^2 - 2(2)$

$= 12$ 1 (2)

(ii) sum of roots $= \frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

$= \frac{\alpha^3 + \beta^3}{\alpha\beta}$ 1

$= \frac{(\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)}{\alpha\beta}$

$= \frac{(-4)(12 - 2)}{2}$

$= -20$ 1

product of roots $= \frac{\alpha^2}{\beta} \times \frac{\beta^2}{\alpha}$

$= \alpha\beta$

$= 2$

$\therefore$ quadratic is 1 (3)

$x^2 + 20x + 2 = 0$

c)(i) limiting sum will exist for $-1 < x < 1$ (1)

(ii) $1 + 2x + 3x^2 + 4x^3 + \dots$

$= (1 + x + x^2 + x^3 + \dots)$

$+ (x + x^2 + x^3 + x^4 + \dots)$

$+ (x^2 + x^3 + x^4 + x^5 + \dots)$

$+ (x^3 + x^4 + x^5 + x^6 + \dots)$

$= \frac{1}{1-x} + \frac{x}{1-x} + \frac{x^2}{1-x} + \frac{x^3}{1-x} + \dots$

$= \frac{1 + x + x^2 + x^3 + \dots}{1-x}$ 1

$= \frac{\frac{1}{1-x}}{1-x}$

$= \frac{1}{(1-x)^2}$ 1 (3)